Journal of Basic and Applied Engineering Research
p-ISSN: 2350-0077; e-ISSN: 2350-0255; Volume 2, Issue 23; October-December, 2015, pp. 2052-2058

http://www.krishisanskriti.org/Publication.html

"Statistical Structures in Air Pollutants Time Series in an Urban Atmosphere"

Sanyogita Singh

School of Environmental sciences, JNU
E-mail: sanyo1610@gmail.com

Abstract—*The present study explores the probability distribution pattern in secondary pollutants such as Ozone and PAN as well as its precursors namely NO_2 and NOx. Pearsons' types of distribution have been examined on the observed data of pollutants at four different sites of Delhi. The results indicate that during winter season all the pollutants follow Type-1 distribution.*

Keywords*: Pearsons' distribution, PAN, Ozone.*

1. INTRODUCTION

The air pollutants generated through the combustion processes are often found to be hazardous for human population. Most of these pollutants react among themselves in the presence of other atmospheric/meteorological variables and form secondary pollutants such as O_3 and Peroxyacetyl nitrate (PAN). The concentrations of the air pollutants are primarily governed by the changes in the emission patterns or through fluctuations in meteorological variables. These fluctuations causes stochasticity in the air pollutants concentration time series and their behavior became more similar to random fluctuations. The modeling studies based on probability distribution function (PDF) were widely applied for characterizing a random fluctuation (Gokhale and Khare, 2006). The PDF of any data series give useful information about the pollutants, which can be used in an organized and more efficient manner. The statistical distributional analysis is also very important in order to remove the inherent noise present in the series mainly due to measurement or instrument error. Air pollution data series are realization of different dynamical and random processes prevalent in the ambient atmosphere. The interactions among different dynamical and random processes cause different orders of nonlinearity and complexity in the series. The statistical distributional models are capable to explain the stochastic variability present in the series. The type of distributional form and values of the associated parameters, related to a number of complex influencing variables, including pollutants and source types, averaging times, emission patterns, prevailing meteorology and topography (Jakeman et al., 1988). Several statistical distribution models have proven to be useful tools in representing pollutant concentration data (Jakeman and Taylor, 1989).

To identify the distributional form Larsen (1969, 1771) and Ott and Mage (1976) used graphical technique to all types of air pollutants concentration data. This scheme is found to be more suitable for small data sets. Later Kolmogorov and Smirnov (K-S) statistics were applied by Kalpasanov and Kurchatova (1976) to examin the statistical distribution form of the different chemical species. Tong and De Pietro (1977) applied chi-square test to confirm log normal distribution for sulfate particulate concentration data. Ott et al., (1979) identify log normal model for CO concentration data using chi-square and KS statistics. Tsukatani and Shigemitsu (1980) applied for the first time the pearson distribution system to the air pollutants time series and identified pearson type VI as best fit distribution in comparison to log normal distribution. Taylet et al. (1985) identify and evaluated the relevence of gooodness of fit test for different air pollutants such as SPM , O_3, CO, NOx, SO_2 and NO_2. Hsin-Chung and Guor-Cheng (2003) applied the method of least squares to estimate the parameters of three theoretical distributions for the respirable particultae matter (PM10) concentration data. In india Gokhale and Khare (2006) examined the statistical behaviour of CO concentration data in urban environment. They obseved that Log -logistic distribution model best fit for the CO concentration data in urban environment of Delhi. Till data date no such study has been carried out on other air pollutants data such as CO, NOx and PAN and also on meteorological parameters in India. To have better insight about the stochastic variability of pollutant time series, Pearsons system of distribution have been examined for O_3, NOx and PAN at 4 different locations in Delhi.

Section 2 deals with the basic theory of pearsonian system of distributions. Results of distribution fittings are presented in section 3.

2. BASICS OF PEARSON'S DISTRIBUTION SYSTEM

The Pearson distribution is a family of continuous probability distributions. It was first published by Karl Pearson in 1895 and subsequently extended by him in 1901 and 1916 in a series of articles on biostatistics. Pearsonion distribution system has taken every valid combination of mean, standard deviation, skewness, and kurtosis into account to model statistical frequency distribution function [Pearson, 1885; 1901; 1916]. Rhind (1909) devised a simple way of visualizing the parameter space of the Pearson system, which was subsequently adopted by Pearson (1916). The Pearson distribution types are characterized by two quantities commonly referred to as β_1 and β_2. The first is the square of the skewness: $\beta_1 = \gamma_1^2$ where γ_1 is the skewness, or third standardized moment. The second is the traditional kurtosis, or fourth standardized moment: $\beta_2 = \gamma_2 + 3$. The diagram below shows which Pearson type a given concrete distribution (identified by a point (β_1, β_2) belongs to.

Fig. 4.1: Diagram of the Pearson system, showing distributions of types I, III, VI, V, and IV in terms of β_1 (squared skewness) and β_2 (traditional kurtosis)

3. DEFINITION

A Pearson density p is defined to be any valid solution to the differential equation (Pearson, 1895).

$$\frac{p'(x)}{p(x)} + \frac{a + x - \lambda}{b_2(x-\lambda)^2 + b_1(x-\lambda) + b_0} = 0. \qquad (1)$$

With:

$$b_0 = \frac{4\beta_2 - 3\beta_1}{10\beta_2 - 12\beta_1 - 18}\mu_2,$$

$$a = b_1 = \sqrt{\mu_2\beta_1}\frac{\beta_2 + 3}{10\beta 2 - 12\beta_1 - 18},$$

$$b_2 = \frac{2\beta_2 - 3\beta_1 - 6}{10\beta 2 - 12\beta_1 - 18}.$$

In equation (1), the parameter a_0 determines a stationary point, and hence under some conditions a mode of the distribution, since

$$p'(a_0) = 0$$

follows directly from the differential equation. Since eq 1 is a linear differential equation with variable coefficients, its solution can be given as

$$p(x) \propto \exp\left(-\int \frac{x - a}{b_2x^2 + b_1x + b_0}\,\mathrm{d}x\right).$$

The integral in this solution simplifies considerably when certain special cases of the integrand are considered. Pearson (1895) distinguished two main cases, determined by the sign of the discriminant (and hence the number of real roots) of the quadratic function

$$f(x) = b_2\,x^2 + b_1\,x + b_0. \qquad (2)$$

Types of distribution

4.2.1 The Pearson type I distribution:

The Pearson type I distribution (a generalization of the beta distribution) arises when the roots of the quadratic equation (2) are of opposite sign, that is, $r_1 < 0 < r_2$. Then the solution p is supported on the interval (r_1, r_2). Apply the substitution

$$x = a_1 + y(a_2 - a_1) \qquad \text{where } 0 < y < 1,$$

which yields a solution in terms of y that is supported on the interval (0,1):

$$p(y) \propto \left(\frac{a_1 - a_2}{a_1}\,y\right)^{(-a_1+a)\nu}\left(\frac{a_2 - a_1}{a_2}\,(1-y)\right)^{(a_2-a)\nu}.$$

Regrouping constants and parameters, this simplifies to:

$$p(y) \propto y^{m_1}(1-y)^{m_2},$$

Thus $\frac{x - \lambda - a_1}{a_2 - a_1}$ follows a $beta(m_1 + 1, m_2 + 1)$ with

$$\lambda = \mu_1 - (a_2 - a_1)\frac{m_1 + 1}{m_1 + m_2 + 2} - a_1$$

It turns out that $m_1 > -1 \wedge m_2 > -1$ is necessary and sufficient for p to be a proper probability density function.

The Pearson type II distribution

The **Pearson type II distribution** is a special case of the Pearson type I family restricted to symmetric distributions.

For the Pearson Type II Curve,

$$y = y_0 \left(1 - \frac{x^2}{a^2}\right)^m$$

where

$$x = \sum d^2/2 - (n^3 - n)/12$$

the ordinate, y, is the frequency of $\sum d^2$. The Pearson Type II Curve is used in computing the table of significant correlation coefficients for Spearman's rank correlation coefficient when the number of items in a series is less than 100 (or 30, depending on some sources). After that, the distribution mimics a standard Student's t-distribution. For the table of values, certain values are used as the constants in the previous equation:

$$m = \frac{5\beta_2 - 9}{2(3 - \beta_2)}$$
$$a^2 = \frac{2\mu_2\beta_2}{3 - \beta_2}$$
$$y_0 = \frac{N[\Gamma(2m+2)]}{a[2^{2m+1}][\Gamma(m+1)]}$$

The moments of x used are

$\mu_2 = (n - 1)[(n^2 + n) / 12]^2$

$$\beta_2 = \frac{3(25n^4 - 13n^3 - 73n^2 + 37n -}{25n(n+1)^2(n-1)}$$

The Pearson type III distribution

$$\lambda = \mu 1 + \frac{b_0}{b_1} - (m+1)b_1$$
$$b_0 + b_1(x - \lambda)$$

follows a :

$$gamma(m+1, b_1^2)$$

Pearson type III distribution is similar to gamma distribution and chi-square distribution.

The Pearson type IV distribution

If the discriminant of the quadratic function (2) is negative ($b_1^2 - 4b_2b_0 < 0$), it has no real roots. Then defining,

$$y = x + \frac{b_1}{2\,b_2}$$

and

$$\alpha = \frac{\sqrt{4\,b_2\,b_0 - b_1^2}}{2\,b_2}.$$

Observe that α is a well-defined real number and $\alpha \neq 0$, because by assumption $4b_2b_0 - b_1^2 > 0$ and therefore $b_2 \neq 0$. Applying these substitutions, the quadratic function (2) is transformed into

$$f(x) = b_2\,(y^2 + \alpha^2).$$

The absence of real roots is obvious from this formulation, because α^2 is necessarily positive.

We now express the solution to the differential equation (1) as a function of y:

$$p(y) \propto \exp\left(-\frac{1}{b_2}\int \frac{y - \frac{b_1}{2\,b_2} - a}{y^2 + \alpha^2}\,\mathrm{d}y\right).$$

Pearson (1895) called this the "trigonometrical case", because the integral

$$\int \frac{y - \frac{2b_2 a + b_1}{2b_2}}{y^2 + \alpha^2} \, \mathrm{d}y = \frac{1}{2} \ln(y^2 + \alpha^2) - \frac{2b_2 a + b_1}{2b_2 \alpha} \arctan\left(\frac{y}{\alpha}\right) + C_0$$

involves the inverse trigonometic arctan function. Then

$$p(y) \propto \exp\left[-\frac{1}{2b_2} \ln\left(1 + \frac{y^2}{\alpha^2}\right) - \frac{\ln\alpha}{2b_2} + \frac{2b_2 a + b_1}{2b_2^2 \alpha} \arctan\left(\frac{y}{\alpha}\right) + C_1\right]$$

Finally, let

$$m = \frac{1}{2b_2}$$

and

$$\nu = -\frac{2b_2 a + b_1}{2b_2^2 \alpha}$$

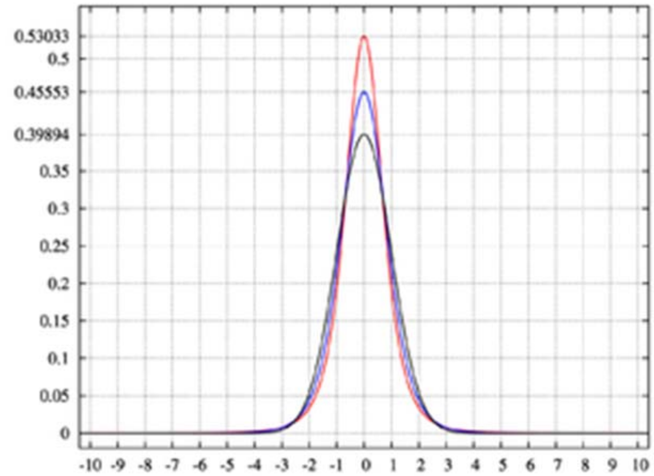
Applying these substitutions, we obtain the parametric function:

$$p(y) \propto \left[1 + \frac{y^2}{\alpha^2}\right]^{-m} \exp\left[-\nu \arctan\left(\frac{y}{\alpha}\right)\right]$$

This unnormalized density has support on the entire real line. It depends on a scale parameter $\alpha > 0$ and shape parameters $m > 1/2$ and ν. One parameter is lost when one chooses to find the solution to the differential equation (1) as a function of y rather than x. Therefore a fourth parameter, namely the location parameter λ is reintroduced. Thus derived the density of the **Pearson type IV distribution** is as given below,

$$p(x) = \frac{\left|\frac{\Gamma\left(m + \frac{\nu}{2}i\right)}{\Gamma(m)}\right|^2}{\alpha \, \mathrm{B}\left(m - \frac{1}{2}, \frac{1}{2}\right)} \left[1 + \left(\frac{x-\lambda}{\alpha}\right)^2\right]^{-m} \exp\left[-\nu \arctan\left(\frac{x-\lambda}{\alpha}\right)\right].$$

The normalizing constant involves the complex Gamma function (Γ) and the Beta function (B).



The Pearson type V distribution

$$\lambda = \mu_1 - \frac{a - C_1}{1 - 2b_2}$$

$$x - \lambda$$

follows a :

$$inversegamma(\frac{1}{b_2} - 1, \frac{a - C_1}{b_2})$$

Pearson type V distribution is similar inverse-gamma distribution.

The Pearson type VI distribution

$$\lambda = \mu_1 + (a_2 - a_1)\frac{m_2 + 1}{m_2 + m_1 + 2} - a_2$$

$$\frac{x - \lambda - a_2}{a_2 - a_1}$$

follows a :

$$betaprime(m_2 + 1, -m_2 - m_1 - 1)$$

Pearson type VI distribution is similar to beta prime distribution, F-distribution.

The Pearson type VII distribution

Plot of Pearson type VII densities with $\lambda = 0$, $\sigma = 1$, and: $\gamma_2 = \infty$ (red); $\gamma_2 = 4$ (blue); and $\gamma_2 = 0$ (black)

The shape parameter ν of the Pearson type IV distribution controls its skewness. If we fix its value at zero, we obtain a symmetric three-parameter family. This special case is known as the **Pearson type VII distribution** (Pearson 1916).

Its density is

$$p(x)=\frac{1}{\alpha\,\mathrm{B}\left(m-\frac{1}{2},\frac{1}{2}\right)}\left[1+\left(\frac{x-\lambda}{\alpha}\right)^2\right]^{-m},$$

where B is the Beta function.

An alternative parameterization (and slight specialization) of the type VII distribution is obtained by letting

$$\alpha=\sigma\sqrt{2m-3},$$

which requires $m > 3/2$. This entails a minor loss of generality but ensures that the variance of the distribution exists and is equal to σ^2. Now the parameter m only controls the kurtosis of the distribution. If m approaches infinity as λ and σ are held constant, the normal distribution arises as a special case:

$$\lim_{m\to\infty}\frac{1}{\sigma\sqrt{2m-3}\,\mathrm{B}\left(m-\frac{1}{2},\frac{1}{2}\right)}\left[1+\left(\frac{x-\lambda}{\sigma\sqrt{2m-3}}\right)^2\right]^{-m}$$

$$=\frac{1}{\sigma\sqrt{2}\,\Gamma\left(\frac{1}{2}\right)}\times\lim_{m\to\infty}\frac{\Gamma(m)}{\Gamma\left(m-\frac{1}{2}\right)\sqrt{m-\frac{3}{2}}}\times\lim_{m\to\infty}\left[1+\frac{\left(\frac{x-\lambda}{\sigma}\right)^2}{2m-3}\right]^{-m}$$

$$=\frac{1}{\sigma\sqrt{2\pi}}\times 1\times\exp\left[-\frac{1}{2}\left(\frac{x-\lambda}{\sigma}\right)^2\right]$$

This is the density of a normal distribution with mean λ and standard deviation σ.

It is convenient to require that $m > 5/2$ and to let

$$m=\frac{5}{2}+\frac{3}{\gamma_2}.$$

This is another specialization, and it guarantees that the first four moments of the distribution exist. More specifically, the Pearson type VII distribution parameterized in terms of (λ, σ, γ_2) has a mean of λ, standard deviation of σ, skewness of zero, and excess kurtosis of γ_2.

Student's t-distribution

Student's *t*-distribution is a special case of Pearson type VII distribution and it arises as the result of applying the following substitutions to its original parameterization:

$$\lambda=0,$$

$$\alpha=\sqrt{\nu},\ \text{and}$$

$$m=\frac{\nu+1}{2},$$

where $\nu > 0$. It may be observed that the constraint $m > 1/2$ is satisfied. The density of this restricted one-parameter family is

$$p(x)=\frac{1}{\sqrt{\nu}\,\mathrm{B}\left(\frac{\nu}{2},\frac{1}{2}\right)}\left[1+\frac{x^2}{\nu}\right]^{-\frac{\nu+1}{2}},$$

which is easily recognized as the density of Student's *t*-distribution.

4. RESULTS AND DISCUSSION

For modeling studies, the identification of the probability distribution function of an air pollutant system is quite useful in understanding the underlying phenomenon governing the system. Till date rather limited numbers of studies have been carried out on the statistical characterization of air polluatnts time series. In most of these studies log normal distributions were found to be best fit with observed data of the air pollutants concentrations (Gokhale and Khare, 2006; Kan and Chen, 2004).

Although in noise pollution studies, different Pearsons' family of distributions have been examined in detail (Tang and Au, 1999 and Tang, 2002), no such attempts have been made to try and see whether Pearsons' family of distributions also characterize air polluatnts time series. Since in the case of air pollutants, their levels, as in case of noise pollution, are also goverened by the types of sources and the prevailing atmospheric conditions. Pearson family of distributions has been examined for the air pollution data monitored at 4 different locations in Delhi.Table 3.1 shows the model fitting results for the seven sites in Delhi. It is pertinent to mention that the data were collected at JNU, Ashok Vihar and Dwarka during winter season. It is evident from the table 3.1 that all the pollutants at most of the sites follow Pearson type I distribution. Pearsons type distribution are similar to a β-distribution.

Distribution was also found to characterize the noise pollution levels in the studies of Tangand Au, 1999 and Tang, 2002. Since Pearson type I distribution is the most commonly observed distribution in the urban atmosphere, it may have to do with the fact that the main source of noise or air pollutants is the vehicular traffic. All the sites are either commercial locations or residential locations. Hence, changes in the emission pattern alter the statistical signature of the pollutants here.

It may conclude that Pearson type I distribution is the most suitable distribution in case of urban air pollutants such as O_3, NO_2, NOx and PAN.

Table 4. 1: Distribution parameters for different pollutants at different locations in Delhi.

		μ1	μ2	μ3	μ4	B1	B2	k	Type
JNU_Winter	*PAN*	1.31	1.92	7.38E+000	3.72E+001	7.69	10.07	-2.139	I
	NO2	18.48	101.82	-1.28E+002	2.74E+004	0.015	2.64	-0.015	I
	NOx	19.41	105.51	-2.34E+002	3.06E+004	0.046	2.74	-0.055	I
	O3	0.0066	7.6851E-05	6.55E-007	1.41E-008	0.945	2.38	-0.251	I
Dwarka_winter	*PAN*	1.41	3.36	1.96E+001	1.71E+002	10.126	15.174	-4.5739	I
	NO2	8.89	62.25	5.66E+002	1.35E+004	1.3269	3.4933	-0.46749	I
	NOx	12.91	3585.5	3.69E+006	3.89E+009	296.05	302.89	-74.256	I
	O3	0.003	3.5066E-05	3.59E-007	5.85E-009	2.9893	4.7544	-0.8191	I
Ashok Vihar_Winter	*PAN*	0.68874	0.11467	3.42E-002	4.76E-002	0.77722	3.6174	-0.63908	I
	NO2	9.3315	17.988	4.34E+001	7.37E+002	0.32393	2.2768	-0.11462	I
	NOx	10.138	20.74	5.57E+001	9.89E+002	0.34741	2.2994	-0.1224	I
	O3	0.041526	0.0004833	1.56E-006	4.84E-007	0.021582	2.071	-0.0087794	I
Delhi Secretariat_winter	*PAN*	0.83999	0.69326	1.90E+000	7.38E+000	10.804	15.348	-4.0666	I
	NO2	11.52	38.154	2.42E+002	4.66E+003	1.0536	3.2023	-0.38102	I
	NOx	12.298	46.803	3.94E+002	8.30E+003	1.5138	3.7876	-0.55407	I
	O3	0.005125	6.9102E-05	1.02E-006	2.53E-008	3.1416	5.2896	-0.94925	I

5. CONCLUSION

The present study explores the probability distribution pattern in secondary pollutants such as ozone and PAN as well as its precursors namely NO2 and NOx. At each of the four sites, namely, Dwarka, Delhi sec, JNU and Ashok Vihar, the pearson's type of distribution were estimated based on the observed data. Pearson's distribution indicates towards the physical processes that govern the system. In other words, it tells the nature of sources of particular pollutants. The results indicate that during winter season all the pollutants follow Type-1 distribution, which is common on urban studies more specifically in case of noise pollution. Pearson's Type-1 distribution are similar in nature to Beta distribution and presents an urban processes since major source of pollutants is traffic for noise and air pollution both, hence it is in the confirmity with the earlier studies carried out elsewhere in the world.

REFERENCES

[1] Agrawal, M., Singh, M., Singh, B., Rajput, M., Marshall, F., and Bell, J.N.B. (2003): "Effect of air pollution on peri-urban agriculture: a case study". Environmental Pollution 126, 3, 323-329.

[2] Ahammed, Y.N., Reddy, R.R., RamaGopal, K., Narasimhulu, K., Bababasha, D., Siva shankar reddy, L., and Rao, T.V.R. (2006): "Seasonal variation of the surface ozone and its precursor gases during 2001-2003, measured at Anantpur, semi- arid site in India". Atmospheric Research, 80, 151-164.

[3] Akaike, H. (1974): "A new look at the statistical model identification, IEEE Transactions on Automatic Control" 19(6): 716-723.

[4] Andersen, C. P. (2003): "Source-sink balance and carbon allocation below ground in plants exposed to ozone." New Phytologist, 157, 213-228.

[5] Aneja, V.P., Agarwal, A., Roelle, P.A., Phillips, S.B., Tong, Q., Watkins, N., Yablonsky, R.(2001): "Measurements and analysis of criteria pollutants in New Delhi, India". Environment International 27, 35-42.

[6] Biene, H. and Krognes, T., (1999): "The seasonal cycle of PAN in the European Arctic". Atmos Envt. 34, 6, 933-940.

[7] Blanchard, C.L., Fairley, D., (2001): "Spatial mapping of VOCs and NOx- limitation of ozone formation in central California". Atmos Envt 35, 3861-3873.

[8] Davidson, J.E.H., Hendry, D.F., Sbra, F., Yeo, S.(1978): " Econometric modeling of aggregate time series Relationships between consumers' Expenditure and Income in the United Kingdom." Economic Journal, 88: 661-92.

[9] Gaffney et al. (1999): "Measurements of PANs in Mexico City: implications for megacity air quality impacts on regional scales". 33, 5003-5012.

[10] Gaffney et al., (1999): "Measurements of PANS in Mexico City: implications for megacity air quality impacts in regional scales, Atmospheric Environment." 33, 5003-5012.

[11] Gaffney, J.S., and Marley, N.A. (1997): "Potential air quality effects of using Ethanol-Gasoline fuel blends: A field study on Albuquerque, New Mexico". Environmental Science and Technology, 31, 11, 3053-3061.

[12] Gaffney, J.S., and Marley, N.A.(1993) Measurements of PAN at a remote site in the Southwestern United States: Tropospheric Implications. Environ. Sci. Technol., 27, 9, 1905-1910.

[13] Hannan, E. J. and Quinn, B.G. (1979): "The determination of the order of an Autoregression." Journal of the Royal Statistical Society, B, 41, 190-195.

[14] Jain, S.L., Arya, B.C., Ahammed, Y. N., Kumar, A., Shukla, D. K., Sinhaand P.R., and Kulkarni, P.S. (2006); "Surface ozone studies at NPL, New Delhi during 1997-2006"; The 2nd Symposium on Asian Ozone Pollution in Eurasian Perspective; Oct 30, 2006; National Physical Laboratory, New Delhi, India.

[15] Jakeman, A.J., Simpson, R.W., Taylor, J. A., (1998): "Modelling distributions of air pollutant concentrations – III: hybrid modeling deterministic- statistical distributions." Atmospheric Environment 22(1), 163-174.

[16] *Pearson (1893): "Contributions to the mathematical theory of evolution" Proceedings of the Royal Society* ***54*** *(326–330): 329–333.*

[17] *Pearson, K. (1895): "Contributions to the mathematical theory of evolution, II: Skew variation in homogeneous material." Philosophical Transactions of the Royal Society* ***186****: 343–414.*

[18] Pearson, Karl *(1901). "Mathematical contributions to the theory of evolution, X: Supplement to a memoir on skew variation." Philosophical Transactions of the Royal Society A* ***197*** *(287–299): 443–459*

[19] *Pearson, Karl (1916). "Mathematical contributions to the theory of evolution, XIX: Second supplement to a memoir on skew variation." Philosophical Transactions of the Royal Society A* ***216*** *(538–548): 429–457.*

[20] *Rhind, A. (July–October 1909). "Tables to facilitate the computation of the probable errors of the chief constants of skew frequency distributions." Biometrika* ***7****(1/2): 127–147.*

[21] Gokhale and Khare (2006): "Statistical behavior of carbon monoxide from vehicular exhausts in urban environments", Environmental Modelling and Software 22(4):526-535.

[22] Kan and Chen (2004): "Exposures and health outcomes from outdoor air pollutants in China, Toxicology,198(1-3):291-300.

[23] Tang and Au (1999): "Statistical structures of indoor traffic noise in a high-rise city", J. Acoust. Soc. Am. **106**, 3415.

[24] Tang et al. (2002): "On the addition of two incoherent unsteady noise records of similar statistical structures", Applied Acoustics, 68(8),829-848.